Chapter – 3 Properties of Gases

* **Vapour** :- state of substance in which evaporation from it liquid state is **incomplete**
  eg. - wet steam

* **Gas** :- state of substance in which evaporation from its liquid state is **complete**
  eg. - H_2, O_2, air etc.

* **Perfect gas** :- Gas that obeys laws of Boyle's & charles under all conditions of temperature & Pressure

→ **Boyle's law** :- Volume of perfect gas is inversely proportional to its absolute pressure when temperature is constant

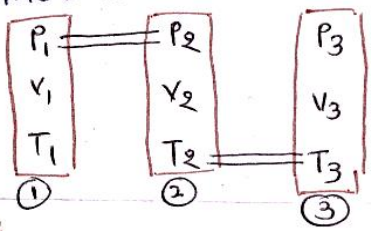
$$V \propto \frac{1}{P} \quad (T = \text{constant}) \Rightarrow PV = C$$

→ **charle's law** :- Volume of perfect gas is directly proportional to its absolute temperature when pressure is constant

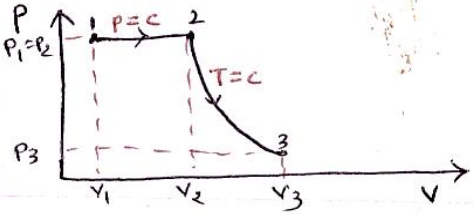
$$V \propto T \quad (P = \text{constant}) \Rightarrow \frac{V}{T} = C$$

→ **Combined Gas law** :-

Process



P-V diagram



→ For process 1-2
According to charle's law

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\therefore \boxed{V_2 = \frac{V_1}{T_1} \cdot T_2} \quad ---(1)$$

→ For process 2-3
According to Boyle's law

$$P_2V_2 = P_3V_3$$

$$\boxed{V_2 = \frac{P_3V_3}{P_2}} \quad ---(2)$$

from eqn ① and ②

$$\frac{V_1}{T_1} \cdot T_2 = \frac{P_3 V_3}{P_2}$$

$$\therefore \quad \frac{P_2 V_1}{T_1} = \frac{P_3 V_3}{T_2} \hookrightarrow$$

$$\therefore \quad \frac{P_1 V_1}{T_1} = \frac{P_3 V_3}{T_3} \quad [\because P_1 = P_2 \text{ \& } T_2 = T_3]$$

$$\therefore \quad \boxed{\frac{PV}{T} = \text{constant}}$$

$$\text{or} \quad \frac{PV}{T} = mR \Rightarrow \boxed{PV = mRT}$$

R = characteristics gas constant unit = kJ/kg·K

[Its value depends on type of gas. for air R = 0.287 kJ/kg·K]

→ Avogadro's law :- All pure gases have same number of molecules per unit volume at same temperature and pressure

We know that, $PV = mRT$

Here, if we take $V = V_0$ [molar volume - volume of 1 mole]

than $m = M$ [molecular mass of gas]

So, $PV_0 = MRT$

but MR is constant for all gases from avogadro's law.

Take $MR = R_0$

$\therefore \quad PV_0 = R_0 T$

for n mole $\boxed{PV = nR_0T}$

R_0 = Universal gas constant unit = kJ/kg·mol k

[Its value is same for all gases & it it 8.314 kJ/kg mol k]

NTP - Normal temp. & pressure = 0°c & 1.01325 bar

STP - Standard temp. & pressure = 25°c & 1.01325 bar

⇒ Relation bet^n C_p & C_v

* Heat added at constant volume

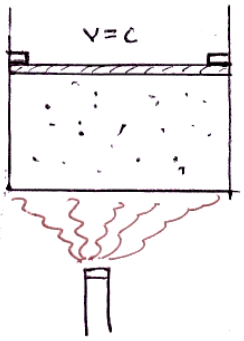
$$Q_v = m \cdot C_v \cdot (T_2 - T_1)$$

Now, from 1st law of thermodynamics

$$Q = \Delta U + W$$

Here $W = 0$ (∵ No movement of piston)

So, $Q = \Delta U \Rightarrow \Delta U = m \cdot C_v (T_2 - T_1)$ -- ①



* Heat added at constant pressure

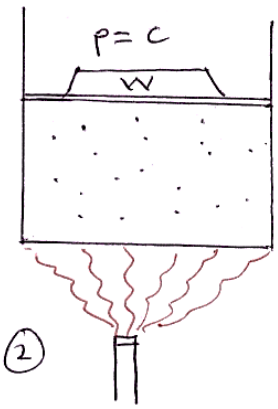
$$Q_p = m \cdot C_p (T_2 - T_1)$$

from 1st law of thermodynamics

$$Q = \Delta U + W$$

Here, $\Delta U = m \cdot C_v (T_2 - T_1)$ (from eqn ①)

and $W = P(V_2 - V_1)$



So, $m \cdot C_p (T_2 - T_1) = m \cdot C_v (T_2 - T_1) + P(V_2 - V_1)$ --②

from characteristics gas eqn

$$PV_1 = mRT_1$$
$$PV_2 = mRT_2$$
$$\therefore PV_2 - PV_1 = mRT_2 - mRT_1$$
$$\therefore P(V_2 - V_1) = mR(T_2 - T_1) \text{ --- } ③$$

from eqn ② and ③

$$m \cdot C_p (T_2 - T_1) = m \cdot C_v (T_2 - T_1) + mR(T_2 - T_1)$$
$$C_p = C_v + R$$

$$\boxed{C_p - C_v = R}$$

Relation betⁿ C_p, C_v, R & γ

	Value for air
C_p = specific heat at constant pressure	1.005 kJ/kg k
C_v = specific heat at constant volume	0.717 kJ/kg k
γ = Adiabatic Index	1.4
R = characteristics gas constant	0.287 kJ/kg·K

$$\boxed{C_p - C_v = R} \qquad \boxed{C_v = \frac{R}{\gamma - 1}}$$

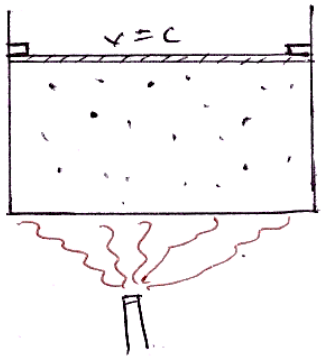
$$\boxed{\frac{C_p}{C_v} = \gamma} \qquad \boxed{C_p = \frac{R \cdot \gamma}{\gamma - 1}}$$

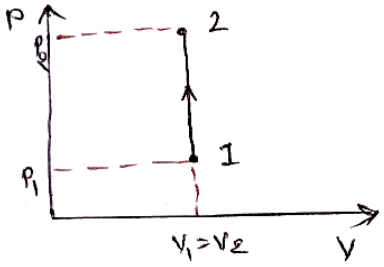
Type of Process

① Flow process – fluid enters system, leave after doing work → e.g. open system

② Non flow process – No mass interaction
e.g. closed system
- (i) constant volume process
- (ii) constant pressure process
- (iii) constant temperature process
- (iv) Adiabatic process
- (v) Polytropic process

① constant volume / isochoric process





(A) PVT relation

we know that

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

If $V_1 = V_2$,

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

or $\boxed{\frac{P}{T} = \text{constant}}$

(B) Work done :-

$W = P\Delta V$ but $\Delta V = 0$ ($\because$ v = constant)

so, $\boxed{W = 0}$

(C) change in Internal energy (ΔU)

$\boxed{\Delta U = m \cdot C_v (T_2 - T_1)}$ [IT WILL BE SAME FOR ALL PROCESS]

(D) Heat transfer (Q)

According to 1st law of thermodynamics

$Q = \Delta U + W = m \cdot C_v (T_2 - T_1) + 0 \Rightarrow \boxed{Q = m \cdot C_v (T_2 - T_1)}$

(E) change in Enthalpy (ΔH)

$\Delta H = H_2 - H_1 = U_2 + P_2V_2 - (U_1 + P_1V_1)$

$= U_2 - U_1 + P_2V_2 - P_1V_1$

=

$$\Delta H = m \cdot C_v (T_2 - T_1) + mR(T_2 - T_1)$$

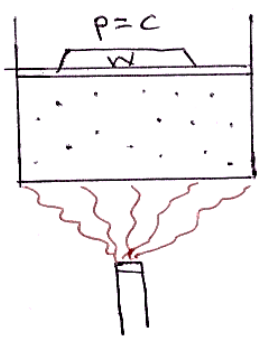
$[\because \Delta U = m C_v (T_2 - T_1)$ & $P_2V_2 - P_1V_1 = mR(T_2 - T_1)]$

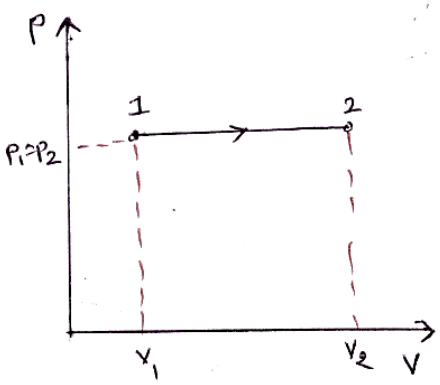
So, $\Delta H = m(T_2 - T_1)[C_v + R]$

$$\boxed{\Delta H = m \cdot C_p (T_2 - T_1)} \quad (\because C_p - C_v = R \Rightarrow C_v + R = C_p)$$

[IT WILL BE SAME FOR ALL PROESS)

(2) constant pressure process / isobaric process





(A) PVT relation

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$P_1 = P_2$, $\frac{V_1}{T_1} = \frac{V_2}{T_2}$

$$\therefore \boxed{\frac{V}{T} = \text{constant}}$$

(B) Work done (W) :-

$$W = P\Delta V$$

$$\boxed{W = P(V_2 - V_1)}$$

(C) change in Internal energy (ΔU) :-

$$\boxed{\Delta U = m \cdot C_v (T_2 - T_1)}$$

(D) Heat Transfer (Q) :-

from 1st law of thermodynamics

$$Q = \Delta U + W = m \cdot C_v (T_2 - T_1) + P(V_2 - V_1)$$

$$= m \cdot C_v (T_2 - T_1) + mR(T_2 - T_1)$$

$[\because P(V_2 - V_1) = mR(T_2 - T_1)]$

$$= m(T_2 - T_1)[C_v + R]$$

$$\boxed{Q = m \cdot C_p (T_2 - T_1)} \quad (\because C_v + R = C_p)$$

(E) change in Enthalpy (ΔH) :-

$$\boxed{\Delta H = m \cdot C_p (T_2 - T_1)}$$ [can be proved as in constant volume process]

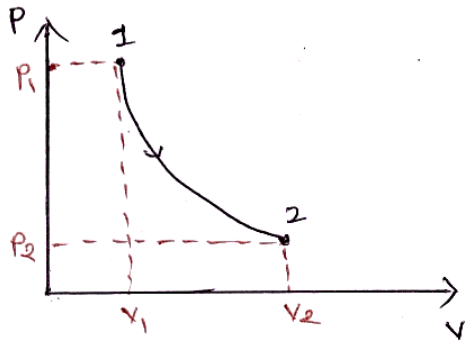
(3) Constant Temperature / Isothermal / Hyperbolic / constant Internal energy process :-

(A) PVT relation :-

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

$T_1 = T_2 \Rightarrow P_1V_1 = P_2V_2$

$\therefore$ $\boxed{PV = \text{constant}}$



(B) Work done (W) :-

$$W = \int_{V_1}^{V_2} P \cdot dV$$

$$= c \int_{V_1}^{V_2} \frac{1}{V} \cdot dV \quad \left[\because PV = c \Rightarrow P = \frac{c}{V}\right]$$

$$= c \left[\ln V\right]_{V_1}^{V_2} \quad \left[\because \int \frac{1}{x} dx = \ln x\right]$$

$$= c \left[\ln V_2 - \ln V_1\right]$$

$$\boxed{W = P_1V_1 \ln V_2/V_1} \quad [\because c = P_1V_1]$$

Or $W = mRT_1 \ln V_2/V_1$

Or $W = P_1V_1 \ln P_1/P_2$

(C) change in Internal Energy (ΔU) :-

$\Delta U = m \cdot C_v (T_2 - T_1)$

$\boxed{\Delta U = 0}$ $(\because T_1 = T_2)$

(D) Heat Transfer (Q) :-

$Q = \Delta U + W = 0 + P_1V_1 \ln V_2/V_1$

$\boxed{Q = P_1V_1 \ln V_2/V_1}$

(E) change in Enthalpy (ΔH) :-

$\Delta H = m \cdot C_p (T_2 - T_1)$

$\boxed{\Delta H = 0}$ $(\because T_1 = T_2)$

(4) Adiabatic process ($Q=0$)

In this process, heat transfer $Q = 0$

→ Law of adiabatic process ($PV^{\gamma} = C$)

According to 1st law of thermodynamics

$$Q = \Delta U + W$$

$$0 = m \cdot c_v \cdot dT + p \cdot dV \quad [Q=0 \text{ for adiabatic process}] \quad \text{---} (1)$$

Now, from characteristic gas equation

$$pV = mRT$$

$$\therefore p\,dV + V\,dp = mR\,dT$$

$$\therefore dT = \frac{p \cdot dV + V\,dp}{mR} \quad \text{---} (2)$$

from eqn (1) and (2)

$$m \cdot c_v \left(\frac{p \cdot dV + V \cdot dp}{mR}\right) + p \cdot dV = 0$$

$$\therefore \frac{c_v \cdot p \cdot dV}{} + c_v \cdot V \cdot dp + \frac{R \cdot p \cdot dV}{} = 0$$

$$\therefore \frac{c_v \cdot p \cdot dV + R \cdot p \cdot dV}{} + c_v \cdot V \cdot dp = 0$$

$$\therefore (c_v + R) \cdot p \cdot dV + c_v \cdot V \cdot dp = 0$$

$$\therefore c_p \cdot p \cdot dV + c_v \cdot V \cdot dp = 0 \quad [\because c_v + R = c_p]$$

deride it by $\frac{c_v \cdot p \cdot V}{}$

so, $$\frac{c_p}{c_v} \frac{dV}{V} + \frac{dp}{p} = 0$$

$$\therefore \gamma \cdot \frac{dV}{V} + \frac{dp}{p} = 0 \quad \left(\because \frac{c_p}{c_v} = \gamma\right)$$

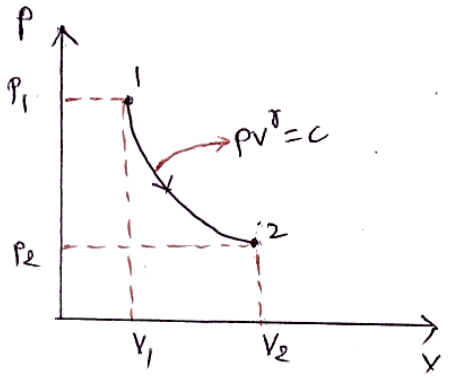
Integrating above eqn

$$\gamma \cdot \ln V + \ln p = \ln C$$

$$\ln V^{\gamma} + \ln p = \ln C$$

$$\therefore \ln pV^{\gamma} = \ln C \Rightarrow \boxed{pV^{\gamma} = C}$$

Adiabatic process ($Q = 0$)



(B) Work done (W) :-

$$W = \int_{V_1}^{V_2} P \cdot dV$$

$$= C \int_{V_1}^{V_2} \frac{1}{V^\gamma} \cdot dV \quad \left[\because PV^\gamma = C, \text{ so } P = \frac{C}{V^\gamma} \right]$$

$$= C \int_{V_1}^{V_2} V^{-\gamma} \cdot dV$$

$$= C \left[\frac{V^{-\gamma+1}}{-\gamma+1} \right]_{V_1}^{V_2} \quad \left[\because \int x^n dx = \frac{x^{n+1}}{n+1} \right]$$

$$= \frac{C}{-\gamma+1} \left[V_2^{-\gamma+1} - V_1^{-\gamma+1} \right]$$

$$= \frac{1}{-\gamma+1} \left[P_2 V_2^\gamma \cdot V_2^{-\gamma+1} - P_1 V_1^\gamma \cdot V_1^{-\gamma+1} \right] \quad \left[\because P_1 V_1^\gamma = P_2 V_2^\gamma = C \right]$$

$$= \frac{1}{-\gamma+1} \left[P_2 V_2 - P_1 V_1 \right]$$

$$\boxed{W = \frac{P_1 V_1 - P_2 V_2}{\gamma - 1}}$$

(C) Change in Internal Energy (ΔU) :-

$$\boxed{\Delta U = m \cdot C_v (T_2 - T_1)} \quad \text{(As usual)}$$

(D) Heat Transfer (Q) :-

$$\boxed{Q = 0} \quad \text{[As per defination !!!]}$$

(E) Change in Enthalpy (ΔH) :-

$$\boxed{\Delta H = m \cdot C_p (T_2 - T_1)} \quad \text{(As usual)}$$

PVT relation for Adiabatic process

we know that

$$P_1 V_1^{\gamma} = P_2 V_2^{\gamma} \quad \text{---} (1) \quad \text{and} \quad \frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} \quad \text{---} (2)$$

$$(1)/(2) \Rightarrow \frac{P_1 V_1^{\gamma}}{P_1 V_1 / T_1} = \frac{P_2 V_2^{\gamma}}{P_2 V_2 / T_2}$$

$$\therefore T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\therefore \frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} \quad \text{---} (3)$$

Now, from eqn (1) $\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^{\gamma}$

$$\therefore \frac{V_2}{V_1} = \left(\frac{P_1}{P_2}\right)^{1/\gamma} \quad \text{---} (4)$$

from (3) and (4)

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left[\left(\frac{P_1}{P_2}\right)^{1/\gamma}\right]^{\gamma-1} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

so

$$\boxed{\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = \left(\frac{P_1}{P_2}\right)^{\gamma-1/\gamma}}$$

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(5) Polytropic process $(PV^n = C)$

$n \rightarrow -\infty$ to $+\infty$

1 to 1.7

(A) PVT relation —

$$\boxed{\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{n-1} = \left(\frac{P_1}{P_2}\right)^{\frac{n-1}{n}}}$$

(B) Work done (W) —

$$\boxed{W = \frac{P_1 V_1 - P_2 V_2}{n-1}}$$

(C) change in Internal energy ~~(dQ)~~ (ΔU)

$$\boxed{\Delta U = m \cdot C_v \cdot (T_2 - T_1)}$$

(D) Heat Transfer (Q)

$$Q = \Delta U + W = \frac{P_1V_1 - P_2V_2}{n-1} + m \cdot C_v (T_2 - T_1)$$

$$= \frac{P_1V_1 - P_2V_2}{n-1} + m \cdot \frac{R}{\gamma - 1} (T_2 - T_1) \quad \left[\because C_v = \frac{R}{\gamma - 1}\right]$$

$$= \frac{P_1V_1 - P_2V_2}{n-1} + \frac{P_2V_2 - P_1V_1}{\gamma - 1} \quad \left[\because mR(T_2 - T_1) = P_2V_2 - P_1V_1\right]$$

$$= P_1V_1 - P_2V_2 \left[\frac{1}{n-1} - \frac{1}{\gamma - 1}\right]$$

$$= P_1V_1 - P_2V_2 \left[\frac{\gamma - 1 - n + 1}{(n-1)(\gamma - 1)}\right]$$

$$= \frac{P_1V_1 - P_2V_2}{n-1} \times \frac{\gamma - n}{\gamma - 1}$$

$$\boxed{Q = W \times \frac{\gamma - n}{\gamma - 1}}$$

(E) change in Enthalpy (ΔH)

$$\boxed{\Delta H = m \cdot C_p (T_2 - T_1)}$$

All processes

$$P_0V^n = C$$

Here, $n = 0 \Rightarrow$ constant Pressure
$n = 1 \Rightarrow$ constant Temp.
$n = \gamma \Rightarrow$ Adiabatic
$n = \alpha \Rightarrow$ constant volume