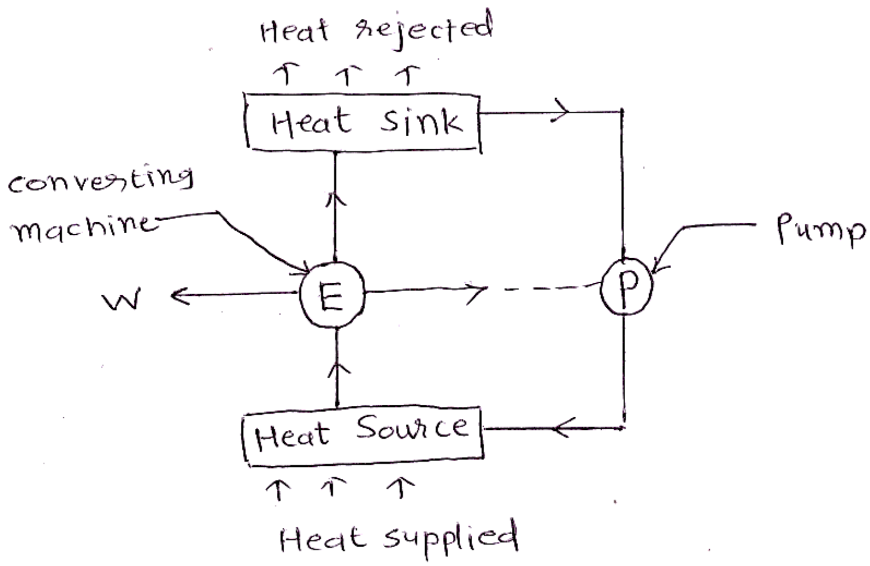
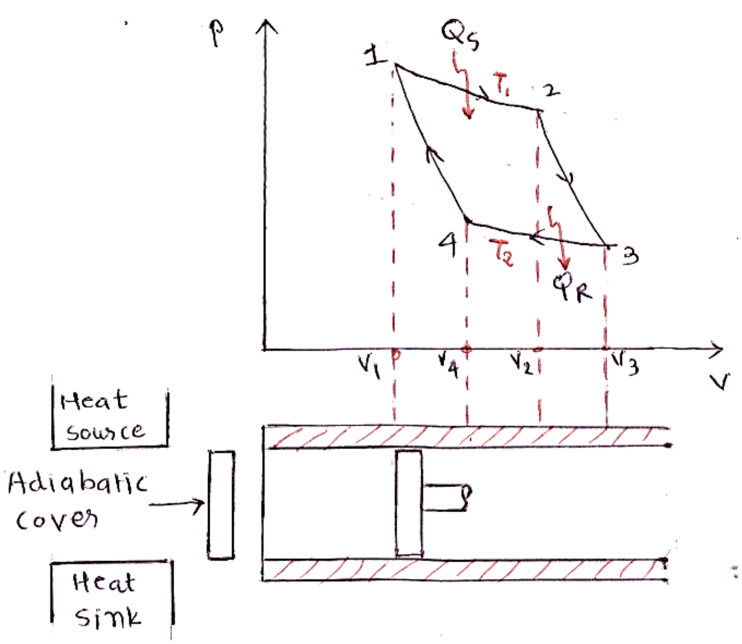
Chapter – 5 Heat Engines

Heat engine :- It is a device which converts chemical energy of fuel into heat energy and this heat energy is used to produce work



⇒ Various Heat engine cycles

① Carnot cycle



Processes

1-2 → Isothermal expansion

2-3 → Adiabatic expansion

3-4 → Isothermal compression

4-1 → Adiabatic compression

→ Efficiency of carnot cycle

* Heat supplied during process (1-2)

$$Q_S = P_1V_1 \ln \frac{V_2}{V_1} = mRT_1 \ln \frac{V_2}{V_1} \quad \text{--- (1)}$$

* Heat rejected during process (3-4)

$$Q_R = P_3V_3 \ln \frac{V_4}{V_3} = mRT_2 \ln \frac{V_4}{V_3} \quad \text{--- (2)}$$

Now, Take r = ratio of expansion = V_2/V_1

= ratio of compression = V_4/V_3

so, from eq^n (1) and (2)

$Q_S = mRT_1 \ln \cdot r$ and $Q_R = mRT_2 \ln \cdot r$

so, Work done $W = Q_S - Q_R$

$= mRT_1 \ln(r) - mRT_2 \cdot \ln(r)$

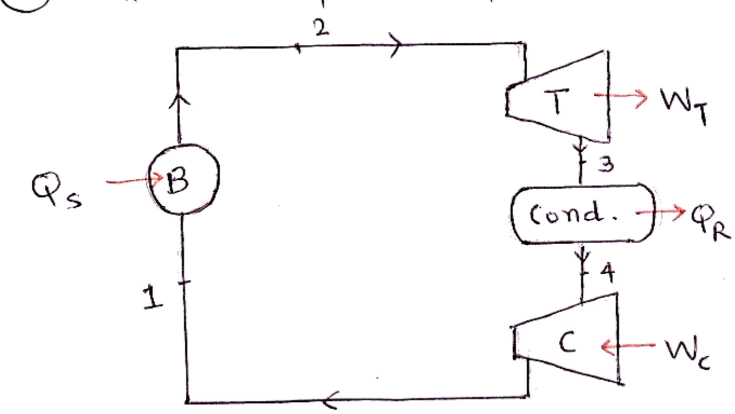
$= mR(T_1 - T_2) \cdot \ln \cdot (r)$

Now, Efficiency $\eta = \frac{\text{output}}{\text{Input}} = \frac{\text{Work done}(W)}{\text{Heat supplied}(Q_S)}$

$$= \frac{mR(T_1 - T_2) \cdot \ln(r)}{mRT_1 \cdot \ln(r)}$$

$$\boxed{\eta = \frac{T_1 - T_2}{T_1}}$$

(2) Carnot vapour cycle

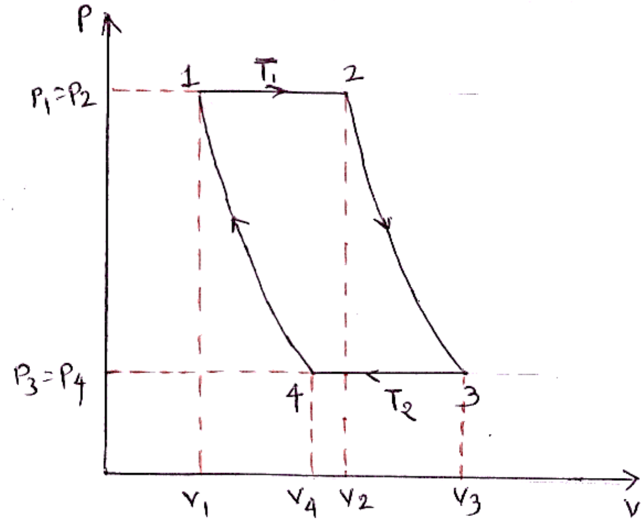


B = Boiler

T = Turbine

Cond. = Condensor

C = compressor



Processes

$1\text{-}2 \rightarrow$ Isothermal expansion

$2\text{-}3 \rightarrow$ Adiabatic expansion

$3\text{-}4 \rightarrow$ Isothermal compression

$4\text{-}1 \rightarrow$ Adiabatic compression

Here, Work

$W_T = h_2 - h_3$

$W_C = h_1 - h_4$

Heat

$Q_S = h - h$

$Q_R = h_3 - h_4$

Now, efficiency $\eta = \dfrac{\text{output}}{\text{Input}} = \dfrac{W}{Q_S}$

$$= \frac{Q_S - Q_R}{Q_S}$$

Now, $Q = T\,\Delta S$ where ΔS = change in entropy

so, $Q_S = T_1(S_2 - S_1)$ and $Q_R = T_2(S_3 - S_4)$

Also, $\Delta S = 0$ For Reversible Adiabatic process

$\therefore$ for process $2-3 \Rightarrow S_2 = S_3$

and for process $4-1 \Rightarrow S_4 = S_1$

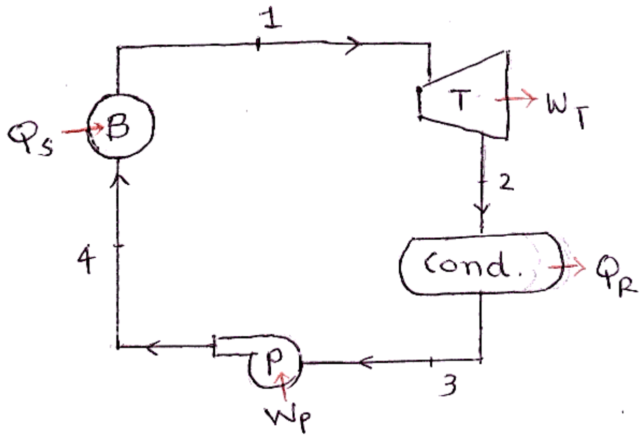
$$\therefore \quad \eta = \frac{Q_S - Q_R}{Q_S}$$

$$= \frac{T_1(S_2 - S_1) - T_2(S_3 - S_4)}{T_1(S_2 - S_1)}$$

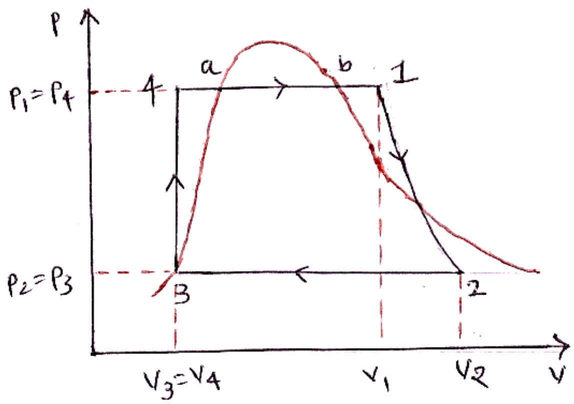
$$= \frac{T_1(S_2 - S_1) - T_2(S_2 - S_1)}{T_1(S_2 - S_1} \quad [\because S_2 = S_3 \ \& \ S_4 = S_1]$$

$$\boxed{\eta = \frac{T_1 - T_2}{T_1}}$$

③ Rankine cycle



B = Boiler
T = Turbine
Cond. = condensor
P = pump



Work

$W_T = h_1 - h_2$

$W_P = h_4 - h_3$

Heat

$Q_S = h_1 - h_4$

$Q_R = h_2 - h_3$

Efficiency $\eta = \dfrac{\text{output}}{\text{Input}} = \dfrac{\text{Work done}}{\text{Heat supplied}}$

$$= \frac{Q_S - Q_R}{Q_S}$$

$$= \frac{(h_1 - h_4) - (h_2 - h_3)}{(h_1 - h_4)} \quad -(1)$$

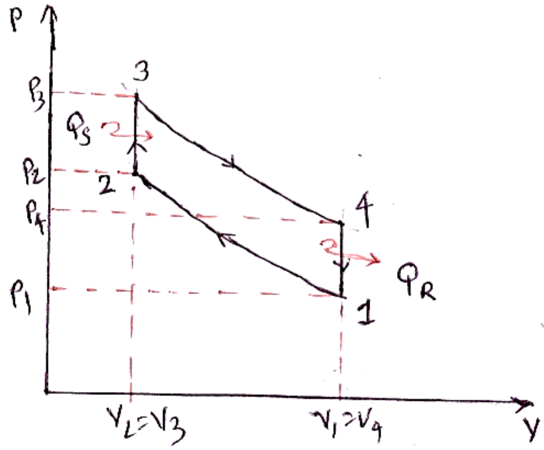
$$\boxed{\eta = 1 - \frac{h_2 - h_3}{h_1 - h_4}}$$

from eq^n (1)

$$\eta = \frac{h_1 - h_4 - h_2 + h_3}{h_1 - h_4} = \frac{(h_1 - h_2) - (h_4 - h_3)}{(h_1 - h_4)}$$

But, $h_4 - h_3$ = pump work (W_P) which is very small and canbe neglected so, $\boxed{\eta = \dfrac{h_1 - h_2}{h_1 - h_4}}$

* Otto cycle :-



Process

- 1-2 → Adiabatic
- 2-3 → constant volume
- 3-4 → Adiabatic
- 4-1 → constant volume

Heat :- for process 2-3 $\quad Q_S = m \cdot C_v (T_3 - T_2)$

for process 4-1 $\quad Q_R = m \cdot C_v (T_4 - T_1)$

Now, Efficiency $\eta = \dfrac{\text{output}}{\text{Input}} = \dfrac{\text{Work done } (W)}{\text{Heat supplied } (Q_s)}$

$$= \frac{Q_S - Q_R}{Q_S}$$

$$= \frac{m \cdot C_v (T_3 - T_2) - m \cdot C_v (T_4 - T_1)}{m \cdot C_v (T_3 - T_2)}$$

$$\eta = 1 - \frac{T_4 - T_1}{T_3 - T_2} \quad \text{--- (1)}$$

Consider

Adiabatic process 1-2 → $\dfrac{T_2}{T_1} = \left(\dfrac{V_1}{V_2}\right)^{\gamma-1}$

$\therefore \; T_2 = T_1 \cdot r^{\gamma-1} \quad (2)$

$$\boxed{\text{compression ratio} \quad r = \frac{V_1}{V_2} = \frac{V_4}{V_3}}$$

Adiabatic process 3-4 → $\dfrac{T_3}{T_4} = \left(\dfrac{V_4}{V_3}\right)^{\gamma-1}$

$\therefore \; T_3 = T_4 \cdot r^{\gamma-1} \quad (3)$

from eq^n (1), (2) and (3)

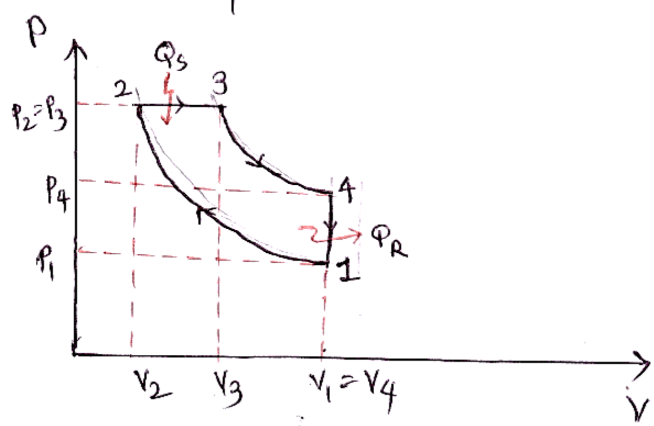
↳

$$\eta = 1 - \frac{T_4 - T_1}{T_4 \cdot r^{\gamma-1} - T_1 \cdot r^{\gamma-1}}$$

$$= 1 - \frac{(T_4 - T_1)}{r^{\gamma-1}(T_4 - T_1)}$$

$$\therefore \boxed{\eta = 1 - \frac{1}{r^{\gamma-1}}}$$

* Diesel cycle :-



Process

1-2 → Adiabatic

2-3 → constant pressure

3-4 → Adiabatic

4-1 → constant volume

Heat Transfer :-

for process 2-3 $\quad Q_s = m \cdot C_p (T_3 - T_2)$

for process 4-1 $\quad Q_R = m \cdot C_v (T_4 - T_1)$

Now, efficiency $\eta = \frac{\text{output}}{\text{Input}} = \frac{\text{Work done}(W)}{\text{Heat supplied}(Q_s)}$

$$= \frac{Q_s - Q_R}{Q_s}$$

$$= \frac{m \cdot C_p (T_3 - T_2) - m \cdot C_v (T_4 - T_1)}{m \cdot C_p (T_3 - T_2)}$$

$$= 1 - \frac{T_4 - T_1}{\gamma (T_3 - T_2)} \quad --- (1)$$

Here, compression ratio, $r = V_1/V_2$

cutoff ratio, $\rho = V_3/V_2$ and

expansion ratio $= V_4/V_3 = \frac{V_4/V_2}{V_3/V_2} = \frac{r}{\rho}$

consider,

Adiabatic process 1-2 → $\frac{T_2}{T_1} = \left(\frac{V_1}{V_2}\right)^{\gamma-1}$

$$T_2 = T_1 \cdot r^{\gamma-1}$$

constant pressure process

2-3 → $\frac{T_3}{T_2} = \frac{V_3}{V_2}$

$$\therefore \quad T_3 = T_2 \cdot \rho$$

$$\therefore \quad T_3 = T_1 \cdot r^{\gamma-1} \cdot \rho$$

Adiabatic process 3-4 → $\frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{\gamma-1}$

$$T_4 = T_3 \cdot \left(\frac{\rho}{r}\right)^{\gamma-1}$$

$$= T_1 \cdot r^{\gamma-1} \cdot \rho \cdot \frac{\rho^{\gamma-1}}{r^{\gamma-1}}$$

$$T_4 = T_1 \cdot \rho^{\gamma}$$

Put the values of T_2, T_3 & T_4 in eq^n 1

$$\therefore \quad \eta = 1 - \frac{T_1 \cdot \rho^{\gamma} - T_1}{\gamma\left[T_1 \cdot r^{\gamma-1} \cdot \rho - T_1 \cdot r^{\gamma-1}\right]}$$

$$= 1 - \frac{T_1\left(\rho^{\gamma}-1\right)}{\gamma \cdot r^{\gamma-1} \cdot T_1\left(\rho-1\right)}$$

$$\boxed{\eta = 1 - \frac{1}{r^{\gamma-1}}\left[\frac{\rho^{\gamma}-1}{\gamma(\rho-1)}\right]}$$